

Geometry I, 2006 : Coursework 1 Solutions

2. (i) [1 mark] The *zero vector* $\underline{0}$ is the vector with zero length and undefined direction. [1 mark] If A is any point, then $\overrightarrow{AA}$ represents $\underline{0}$.

(ii) [4 marks for completely correct proof; 3 marks for nearly correct proof; 2 marks for something reasonable; no marks for nonsense.]

Let $\overrightarrow{AB}$ represent $\underline{u}$. As $\overrightarrow{BB}$ represents $\underline{0}$, we see by the triangle rule that $\overrightarrow{AB}$ represents $\underline{u} + \underline{0}$, and so $\underline{u} + \underline{0} = \underline{u}$.

3. (i) [2 marks] If $\underline{v}$ is a non-zero vector then $-\underline{v}$ is the vector with the same length and opposite direction as $\underline{v}$, and $-\underline{0} = \underline{0}$.

(ii) [4 marks for completely correct proof; 3 marks for nearly correct proof; 2 marks for something reasonable; no marks for nonsense.]

In the lecture notes, it is observed from the definitions of negation and of scalar multiplication that $(-1)\underline{v} = -\underline{v}$, and it is proved that $\alpha(\beta\underline{v}) = (\alpha\beta)\underline{v}$ for all real numbers α, β and all vectors $\underline{v}$.

Thus $(-\alpha)\underline{v} = ((-1)\alpha)\underline{v} = (-1)(\alpha\underline{v}) = -(\alpha\underline{v})$.

4. (i) [2 marks for all correct; 1 mark for 3 correct; no marks otherwise.]

Let $\underline{a}, \underline{b}, \underline{c}, \underline{d}$ be the position vectors of A, B, C, D , and $\underline{p}, \underline{q}, \underline{r}, \underline{s}$ be the position vectors of P, Q, R, S .

Then $\underline{p} = 1/2(\underline{a} + \underline{b})$, $\underline{q} = 1/2(\underline{b} + \underline{c})$, $\underline{r} = 1/2(\underline{c} + \underline{d})$, and $\underline{s} = 1/2(\underline{d} + \underline{a})$.

(ii) [6 marks for completely correct proof; 5 marks for nearly correct proof; 3 marks for reasonable progress; 1 mark for something non-trivial correct.]

$\overrightarrow{PS}$ represents the vector $\underline{s} - \underline{p}$, and from part (i), $\underline{s} - \underline{p} = 1/2(\underline{d} + \underline{a}) - 1/2(\underline{a} + \underline{b}) = 1/2(\underline{d} - \underline{b})$.

$\overrightarrow{QR}$ represents the vector $\underline{r} - \underline{q}$, and from part (i), $\underline{r} - \underline{q} = 1/2(\underline{c} + \underline{d}) - 1/2(\underline{b} + \underline{c}) = 1/2(\underline{d} - \underline{b})$.

Since $\overrightarrow{PS}$ and $\overrightarrow{QR}$ represent the same vector, it follows (from the definition of parallelogram given in the lecture notes) that $PQRS$ is indeed a parallelogram.

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